Field Theoretical Treatment of E-Plane Waveguide Junctions with Anisotropic Medium

Yu-Yuan Tsai and Abbas S. Omar, *Senior Member, IEEE*

Abstract—**An analysis of E-plane waveguide junctions containing an anisotropic medium is presented. The analysis is based on the equivalence principle and on cavity field expansions. Using the equivalence principle, magnetic surface currents are introduced at the imaginary boundaries chosen between the central region of the junction and the waveguides. The electric displacement $\underline{D}$ in the junction is expressed in terms of a solenoidal set while the magnetic induction $\underline{B}$ is expressed in terms of a solenoidal set and an irrotational set. Matching the tangential magnetic field at the imaginary boundaries leads to a matrix equation, the unknown of which are the amplitudes of the scattered waveguide modes. Using this method, the performance of E-plane waveguide junctions with full-height and partial-height ferrite post is analyzed. The influence of the completeness terms $\underline{G}_{oq}$ on the numerical results of an empty E-plane Y-junction is shown. The numerical results are compared with previously published experimental and theoretical results.**

I. Introduction

The E-plane waveguide junction (e.g. E-plane circulator) is widely used in microwave circuits due to its capability to handle high power. This is because the anisotropic medium is not located at the point of maximum electric field intensity. The theoretical treatment of E-plane waveguide junctions containing anisotropic medium has, to the best of our knowledge, only been carried out by using the point-matching method [1], [2]. In the point-matching method, the propagation constants of the guided waves in the anisotropic medium are numerically calculated by solving the characteristic equations, which contain transcendental functions. In order to obtain the performance of the waveguide junction, the most significant propagation constants have to be looked for first from the calculated propagation constants [2]. In this paper, a method based on the equivalence principle and on cavity field expansions is presented with no need to look for the propagation constants of the guided waves in the anisotropic medium. The performance of E-plane waveguide junctions containing full-height and partial-height ferrite post is calculated. The so-called completeness terms $\underline{G}_{oq}$ have to be included in the irrotational set in order to completely describe the field in the junction. The numerical results are finally compared to earlier experimental and theoretical results.

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The authors are with the Technische Universitat Braunschweig, Institut fur Hochfrequenztechnik, Postfach 33 29, D-3300, Braunschweig, Germany.
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II. Mathematical Formulation

Fig. 1 shows an E-plane waveguide junction containing a partial-height ferrite post. The ferrite post has a relative permittivity ϵ_r and a relative permeability tensor $\overleftrightarrow{\mu}_r$. The diameter and the height of the ferrite post are d and h, respectively. The whole structure is assumed to be lossless and the fields are assumed to have a time dependence exp $(j\omega t)$. The waveguides are assumed to support only one propagating mode (the dominant mode), while all higher-order modes are below cut-off. However this does not mean that the higher-order modes are neglected in the calculation. Rectangular and circular cylindrical coordinate systems (x, y, z) and (r, ϕ, z) are employed with the z-axis coinciding with the junction axis of symmetry.

Using the equivalence principle [3], the imaginary boundaries S_g^i chosen between the central region of the junction and the waveguides are short-circuited and magnetic surface currents $\underline{M}_s^i = \underline{\hat{n}} \times \underline{E}_g^i$ and $-\underline{M}_s^i$ are introduced at both sides of the inserted short-circuits in order to restore the non-vanishing tangential electric fields. $\underline{E}_g^i$ is the electric field of the $i'th$ waveguide ($i = 1, 2, 3$) at the imaginary boundaries S_g^i and $\underline{\hat{n}}$ is the unit normal vector at S_g^i. The central region of the junction is treated as an empty cavity resonator excited by magnetic surface currents which are located at the imaginary boundaries S_g^i, and a volume polarization current replaces the anisotropic medium. Since the electric displacement $\underline{D}$ is divergenceless in this resonator, it can be expressed completely as an expression in terms of a solenoidal set

$$\underline{D} = \epsilon_o \sum_n e_n \underline{E}_n \tag{1}$$

where $\underline{E}_n$ means the electric field of the $n'th$ resonance mode in the empty resonator (with the magnetic surface currents at S_g^i and the anisotropic medium removed), and e_n is the corresponding expansion coefficient. Due to the magnetic surface currents at the imaginary boundaries S_g^i, the magnetic induction $\underline{B}$ in the resonator must be expressed in terms of a solenoidal and an irrotational set:

$$\underline{B} = \mu_o \sum_n h_n \underline{H}_n + \mu_o \sum_n g_n \underline{G}_n + \mu_o \sum_q g_{oq} \underline{G}_{oq} \tag{2}$$

where $\underline{H}_n$ is the magnetic field of the $n'th$ resonance mode in the empty resonator, $\underline{G}_n$ and $\underline{G}_{oq}$ belong to the irrotational set. h_n, g_n, and g_{oq} are the corresponding expansion coefficients.

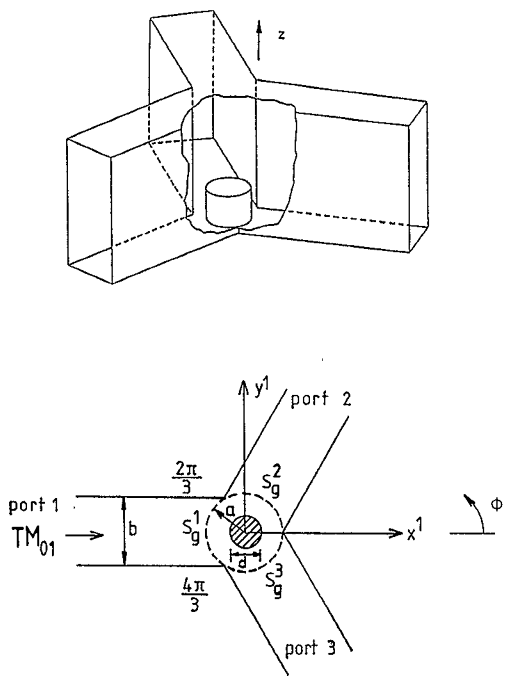

Fig. 1. An E-plane waveguide junction with partial-height ferrite post.

Substituting (1) and (2) into Maxwell's equations, making use of the orthogonal properties of $\{\underline{E}_n\}$, $\{\underline{H}_n\}$, and $\{\underline{G}_n\}$ [4]–[8], of the identity $\nabla \cdot (A \times B) = B \cdot \nabla \times A - A \cdot \nabla \times B$, and of the divergence theorem we obtain

$$\omega_n \epsilon_o \int_v \underline{E}_n^* \cdot \epsilon_r^{-1}(\underline{r}) \sum_m e_m \underline{E}_m \, dV = \omega h_n - j \int_{S_g^i} \underline{M}_s^i \cdot \underline{H}_n^* \, ds \tag{3}$$

$$\omega_n \mu_o \int_v \underline{H}_n^* \cdot \overleftrightarrow{\mu}_r^{-1}(\underline{r}) \sum_m h_m \underline{H}_m \, dV + \omega_n \mu_o \int_v \underline{H}_n^* \cdot \overleftrightarrow{\mu}_r^{-1}(\underline{r}) \sum_m g_m \underline{G}_m \, dV = \omega e_n \tag{4}$$

$$g_n = \frac{j}{\omega} \int_{S_g^i} \underline{M}_s^i \cdot \underline{G}_n^* \, ds \tag{5}$$

where ω is the angular frequency, $*$ denotes complex conjugate, ω_n is the resonant frequency of the $n'th$ mode in the empty resonator, v is the volume of the empty resonator and $\overleftrightarrow{\mu}_r(\underline{r})$ and $\epsilon_r(\underline{r})$ are the relative permeability tensor and relative permittivity in the resonator, respectively, which are functions of the position vector $\underline{r}$.

Considering the excitation by the dominant mode of the waveguide at port 1, the incident electromagnetic field components are given by

$$E_{gy}^{e1}(x^1, z^1) = A_{e1}^1 \sin\left(\frac{\pi}{c} z^1\right) e^{-jk_{e1}^1 x^1} \tag{6}$$

$$H_{gx}^{e1}(x^1, z^1) = \frac{A_{e1}^1}{j\omega\mu_o} \frac{\pi}{c} \sin\left(\frac{\pi}{c} z^1\right) e^{-jk_{e1}^1 x^1} \tag{7}$$

$$H_{gz}^{e1}(x^1, z^1) = \frac{A_{e1}^1 k_{e1}^1}{\omega\mu_o} \cos\left(\frac{\pi}{c} z^1\right) e^{-jk_{e1}^1 x^1} \tag{8}$$

where $k_{e1}^1 = \sqrt{k_o^2 - \gamma_{e1}^1}$, $\gamma_{e1}^1 = \pi/c$, $k_o = \omega\sqrt{\mu_o \epsilon_o}$, and c is the height of the waveguides. Since the incident field varies along the z-direction, and since the ferrite post is not continuous along this direction, all possible waveguide modes are excited. The scattered field can be written as follows:

TM_y modes:

$$E_{gy}^{ei} = \sum_{w=0}^{\infty} \sum_{l=1}^{\infty} B_{wl}^{ei} \cos\left(\frac{\left(1 + \frac{2y^i}{b}\right) w\pi}{2}\right) \cdot \sin\left(\frac{l\pi}{c} z^i\right) e^{jk_{wl}^{ei} x^i} \tag{9}$$

where b is the width of the waveguides and $k_{wl}^{ei} = -j\sqrt{(w\pi/b)^2 + (l\pi/c)^2 - k_o^2}$. The other field components can be derived from

$$\frac{j}{\omega\epsilon_o}\left(k_o^2 + \frac{\partial^2}{\partial y^2}\right) H_{gx}^{ei} = \frac{\partial E_{gy}^{ei}}{\partial z} \tag{10}$$

$$\frac{1}{j\omega\epsilon_o}\left(k_o^2 + \frac{\partial^2}{\partial y^2}\right) H_{gz}^{ei} = \frac{\partial E_{gy}^{ei}}{\partial x} \tag{11}$$

$$E_{gx}^{ei} = \frac{1}{j\omega\epsilon_o} \frac{\partial H_{gz}^{ei}}{\partial y} \tag{12}$$

$$E_{gz}^{ei} = \frac{j}{\omega\epsilon_o} \frac{\partial H_{gx}^{ei}}{\partial y} \tag{13}$$

TE_y modes:

$$H_{gy}^{hi} = \sum_{w=1}^{\infty} \sum_{l=0}^{\infty} B_{wl}^{hi} \sin\left(\frac{\left(1 + \frac{2y^i}{b}\right) w\pi}{2}\right) \cdot \cos\left(\frac{l\pi}{c} z^i\right) e^{jk_{wl}^{hi} x^i} \tag{14}$$

where $k_{wl}^{hi} = -j\sqrt{(w\pi/b)^2 + (l\pi/c)^2 - k_o^2}$.

The components E_{gx}^{hi}, E_{gz}^{hi}, H_{gx}^{hi}, and H_{gz}^{hi} can be derived from

$$\frac{1}{j\omega\mu_o}\left(k_o^2 + \frac{\partial^2}{\partial y^2}\right) E_{gx}^{hi} = \frac{\partial H_{gy}^{hi}(x^i, z^i)}{\partial z} \tag{15}$$

$$\frac{j}{\omega\mu_o}\left(k_o^2 + \frac{\partial^2}{\partial y^2}\right) E_{gz}^{hi} = \frac{\partial H_{gy}^{hi}(x^i, z^i)}{\partial x} \tag{16}$$

$$H_{gx}^{hi} = \frac{j}{\omega\mu_o} \frac{\partial E_{gz}^{hi}(x^i, z^i)}{\partial y} \tag{17}$$

$$H_{gz}^{hi} = \frac{1}{j\omega\mu_o} \frac{\partial E_{gx}^{hi}(x^i, z^i)}{\partial y} \tag{18}$$

$$i = 1, 2, 3$$
$$a = \frac{b}{\sqrt{3}}$$
$$x^i = \frac{b \cos \phi^i}{\sqrt{3}}$$
$$y^i = \frac{b \sin \phi^i}{\sqrt{3}}$$
$$\phi^i = \phi + \frac{(i-1)2\pi}{3}.$$

The resonance modes of the empty cylindrical cavity are either TM to z or TE to z. The TM to z modes ($\underline{E}_n^E$, $\underline{H}_n^E$) are derived from a magnetic vector potential $\underline{A}$ which has a z-component ψ_n^E only [3]:

$$\psi_n^E = \psi_{msq}^E = A_{msq} J_m \left(x_{ms} \frac{r}{a} \right) e^{jm\phi} \cos \left(\frac{q\pi}{c} z \right) \tag{19}$$

where $n = (m, s, q)$:

$$\cdot \begin{cases} m &= \cdots -2, -1, 0, 1, 2, \cdots \\ s &= 1, 2, 3, \cdots \\ q &= 0, 1, 2, \cdots \end{cases}$$

The TE to z modes ($\underline{E}_n^H$, $\underline{H}_n^H$) are determined from an electric vector potential $\underline{F}$ which has a z-component ψ_n^H only [3]:

$$\psi_n^H = \psi_{msq}^H = B_{msq} J_m \left(x'_{ms} \frac{r}{a} \right) e^{jm\phi} \sin \left(\frac{q\pi}{c} z \right) \tag{20}$$

where $n = (m, s, q)$:

$$\cdot \begin{cases} m &= \cdots -2, -1, 0, 1, 2, \cdots \\ s &= 1, 2, 3, \cdots \\ q &= 1, 2, \cdots \end{cases}$$

The irrotational modes are defined by [8]

$$p_n \underline{G}_n = \underline{\nabla} \psi_n \tag{21}$$

$$\nabla^2 \psi_n + p_n^2 \psi_n = 0 \tag{22}$$

$$\frac{\partial \psi_n}{\partial n} = 0 \text{ on the surface of the resonator} \tag{23}$$

$$\underline{\nabla} \times \underline{G}_n = 0 \tag{24}$$

p_n is the eigenvalue of the $n'th$ solution.

The components of the irrotational set $\underline{G}_n$ can be obtained from the scalar function ψ_n

$$\psi_n = \psi_{msq} = J_m \left(x'_{ms} \frac{r}{a} \right) e^{jm\phi} \cos \left(\frac{q\pi}{c} z \right) \tag{25}$$

which satisfies (22), (23), and (24):

$$G_{nr} = G_{msq;r} = D_{msq} \frac{x'_{ms}}{a} J'_m \left(x'_{ms} \frac{r}{a} \right) \cdot e^{jm\phi} \cos \left(\frac{q\pi}{c} z \right) \tag{26}$$

$$G_{n\phi} = G_{msq;\phi} = D_{msq} \frac{jm}{r} J_m \left(x'_{ms} \frac{r}{a} \right) \cdot e^{jm\phi} \cos \left(\frac{q\pi}{c} z \right) \tag{27}$$

$$G_{nz} = G_{msq;z} = -D_{msq} \frac{q\pi}{c} J_m \left(x'_{ms} \frac{r}{a} \right) \cdot e^{jm\phi} \sin \left(\frac{q\pi}{c} z \right) \tag{28}$$

where $n = (m, s, q)$:

$$\cdot \begin{cases} m &= \cdots -2, -1, 0, 1, 2, \cdots \\ s &= 1, 2, 3, \cdots \\ q &= 0, 1, 2, \cdots \end{cases}$$

A_{msq}, B_{msq}, and D_{msq} can be determined from the normalization of $\{\underline{E}_n\}$, $\{\underline{H}_n\}$, and $\{\underline{G}_n\}$, respectively.

x'_{ms} is the $s'th$ root of $J'_m(\cdot) = 0$ at $r = a$. The root $x'_{ms} = 0$ is generally not considered as in the case of determining the TE-modes of a cylindrical cavity [3], [8]. However, to expand the magnetic induction $\underline{B}$ in the cylindrical resonator in our case the terms $\underline{G}_{oq}$ which correspond to terms with $x'_{ms} = 0$ have to be included for completeness. These terms are constant on the r-ϕ-plane and have only a non-zero z-component

$$\underline{G}_{oq} = \underline{G}_{ooq} \underline{\hat{u}}_z = -D_{ooq} \sin \left(\frac{q\pi}{c} z \right) \underline{\hat{u}}_z; \quad q = 1, 2, \cdots \tag{29}$$

D_{ooq} can be determined from the normalization of $\{\underline{G}_{oq}\}$.

From a mathematical point of view, these terms have the same meaning as the constant term in a one-dimensional Fourier series (our cases are two-dimensional in r and ϕ). The theory of Fourier series is widely known and the necessity of the constant term in a one-dimensional Fourier series has already been discussed (e.g. in [9] and [10]), therefore it will not be repeated here.

These terms $\underline{G}_{oq}$ satisfy orthogonality relations

$$\mu_o \int_v \underline{G}_{oq} \cdot \underline{G}_n^* \, dV = 0,$$
$$\mu_o \int_v \underline{G}_{op} \cdot \underline{G}_{oq}^* \, dV = \delta_{pq} \tag{30}$$

which can easily be verified by carrying out the integration. δ_{pq} is the Kronecker delta.

Note: The terms $\underline{G}_{oq}$ are not the zero frequency mode G_o (static magnetic field) discussed in [5]–[8]. The zero frequency mode cannot exist in the cylindrical resonators discussed in this paper which belong to the type 1 cavity defined in [8].

The continuity of the tangential magnetic field at S_g^i leads to

$$\underline{\hat{n}} \times \underline{H}_g^{i*}|_{S_g^i} = \underline{\hat{n}} \times \underline{H}^*|_{S_g^i} \tag{31}$$

where $\underline{H}_g^i$ is the magnetic field of the $i'th$ waveguide.

Scalar-multiplying (31) by the test vector-functions $\underline{\hat{u}}_\phi e^{ei}_{\lambda\phi}$, $\underline{\hat{u}}_z e^{ei}_{\lambda z}$, $\underline{\hat{u}}_\phi e^{hi}_{\lambda\phi}$, and $\underline{\hat{u}}_z e^{hi}_{\lambda z}$ and integrating over S^i_g, we obtain the following matrix equation:

$$[P]^*\underline{v} = [RR]^*\underline{h} + [SS]^*\underline{g} \tag{32}$$

where $\lambda = (\xi, \nu)$ and

$$e^{ei}_{\lambda\phi} = e^{ei}_{\xi\nu,\phi} = \sin\left(\frac{\nu\pi}{c} z^i\right)\left[\sin\left(\frac{\left(1 + \frac{2y^i}{b}\right)\xi\pi}{2}\right) \cdot \sin\phi^i + \cos\left(\frac{\left(1 + \frac{2y^i}{b}\right)\xi\pi}{2}\right)\cos\phi^i\right] \tag{33}$$

$$e^{ei}_{\lambda z} = e^{ei}_{\xi\nu;z} = \sin\left(\frac{\left(1 + \frac{2y^i}{b}\right)\xi\pi}{2}\right)\cos\left(\frac{\nu\pi}{c} z^i\right) \tag{34}$$

$$e^{hi}_{\lambda\phi} = e^{hi}_{\xi\nu;\phi} = \sin\left(\frac{\left(1 + \frac{2y^i}{b}\right)\xi\pi}{2}\right) \cdot \sin\left(\frac{\nu\pi}{c} z^i\right)\sin\phi^i \tag{35}$$

$$e^{hi}_{\lambda z} = e^{hi}_{\xi\nu;z} = \sin\left(\frac{\left(1 + \frac{2y^i}{b}\right)\xi\pi}{2}\right)\cos\left(\frac{\nu\pi}{c} z^i\right) \tag{36}$$

Equations (3), (4), and (5) can be rewritten in matrix form

$$[C^{ee}]\underline{e} = \omega[I]\underline{h} - [R]\underline{v} \tag{37}$$

$$[C^{hh}]\underline{h} + [C^{hg}]\underline{g} = \omega[I]\underline{e} \tag{38}$$

$$\underline{g} = [S]\underline{v} \tag{39}$$

where $\underline{e}$ is a column vector with elements e_n and $[I]$ is the unity matrix. From (32), (37), (38), and (39) we obtain a matrix equation, the unknown of which are the amplitudes of the waveguide modes

$$[M]\begin{bmatrix}\underline{v}^1\\ \underline{v}^2\\ \underline{v}^3\end{bmatrix} = [0] \tag{40}$$

where

$$[M] = [P]^* + [RR]^*([C^{hh}] - \omega^2[I][C^{ee}]^{-1}[I])^{-1} \cdot (\omega[I][C^{ee}]^{-1}[R] + [C^{hg}][S]) - [SS]^*[S] \tag{41}$$

The matrices $[C^{ee}]$, $[C^{hh}]$, $[C^{hg}]$, $[R]$, $[S]$, $[RR]$, $[SS]$, and $[P]$ have the following form:

$$[C^{ee}] = \begin{bmatrix}[C^{ee}_{HH}] & [C^{ee}_{HE}]\\ [C^{ee}_{EH}] & [C^{ee}_{EE}]\end{bmatrix} \quad [C^{hh}] = \begin{bmatrix}[C^{hh}_{HH}] & [C^{hh}_{HE}]\\ [C^{hh}_{HH}] & [C^{hh}_{EE}]\end{bmatrix} \tag{42}$$

$$[C^{hg}] = \begin{bmatrix}[C^{hg}_{HG}]\\ [C^{hg}_{EG}]\end{bmatrix} \quad [R^i] = \begin{bmatrix}[R^i_{He}] & [R^i_{Hh}]\\ [R^i_{Ee}] & [R^i_{Eh}]\end{bmatrix} \tag{43}$$

$$[S^i] = [[S^i_{Ge}] \;\; [S^i_{Gh}]]$$

$$[RR^i] = \begin{bmatrix}[RR^i_{eH}] & [RR^i_{eE}]\\ [RR^i_{hH}] & [RR^i_{hE}]\end{bmatrix} \tag{44}$$

$$[SS^i] = \begin{bmatrix}[SS^i_{eG}]\\ [SS^i_{hG}]\end{bmatrix} [P^i] = \begin{bmatrix}[P^i_{ee}] & [P^i_{eh}]\\ [P^i_{he}] & [P^i_{hh}]\end{bmatrix}$$

$$i = 1, 2, 3. \tag{45}$$

where the subscript H and E denote the TE_z and TM_z modes of the empty cylindrical resonator, respectively, and h and e denote the TE_y and TM_y modes of the waveguides, respectively. G denotes the irrotational set.

Since the junction is excited by the dominant mode of the waveguide at port 1 with amplitude $A^1_{e1} = 1$, (40) can be rewritten as

$$[M^a]\begin{bmatrix}\underline{v}^{a1}\\ \underline{v}^{a2}\\ \underline{v}^{a3}\end{bmatrix} = -A^1_{e1}\begin{bmatrix}M_{21}\\ M_{31}\\ \vdots\end{bmatrix} \tag{46}$$

where the elements of the column vectors $\underline{v}^{a^i}$ $(i = 1, 2, 3)$ are the amplitudes of the scattered waveguides modes.

The computation time of the method is determined by the computation time required to calculate the elements of the matrices and the computation time required to solve (46). Most elements of the matrices $[R]$, $[S]$, $[RR]$, $[SS]$, and $[P]$ have to be calculated numerically. But if the diameter of the ferrite post is not too large, the required number of waveguide modes is then much less than the required number of expansion functions of the cylindrical resonator. The computation time for the elements of these matrices is thus less critical than that for the elements of the matrices $[C^{ee}]$, $[C^{hh}]$, and $[C^{hg}]$. Fortunately, all the elements of the matrices $[C^{ee}]$, $[C^{hh}]$, and $[C^{hg}]$ can be calculated analytically by using Green's first identity.

IV. Numerical Results

A. Numerical Results for the Empty E-plane Y-junction

In order to show the necessity of the completeness terms $\underline{G}_{oq}$, we consider an empty E-plane Y-junction. The width and height of the waveguides are 10.16 mm and 22.86 mm, respectively. Fig. 2 shows the magnitude of the different S-parameters and the argument of the different ei-

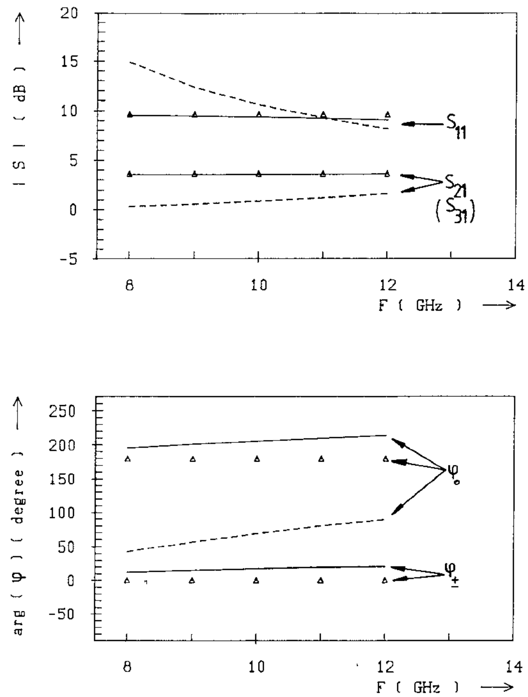

Fig. 2. S-parameters and arguments of the scattering matrix eigenvalues of an empty E-plane Y-junction: ———— values computed with $\underline{G}_{oq}$, ---- values computed without $\underline{G}_{oq}$, △ measured values given by [11].

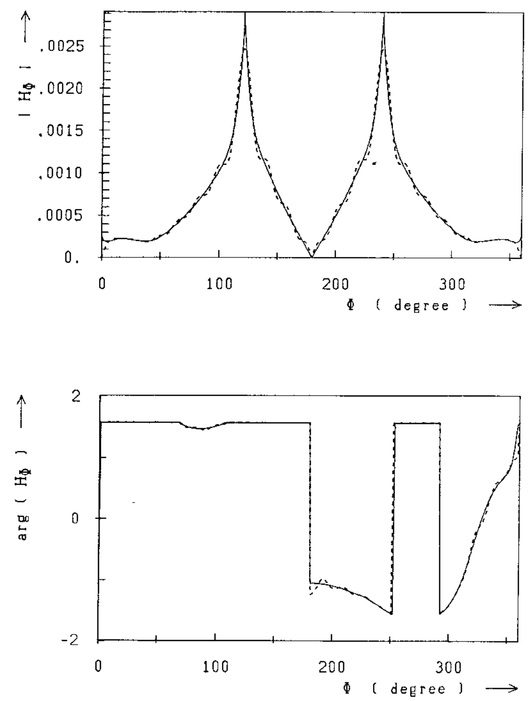

Fig. 3. Distribution of the ϕ-component of the magnetic field at $S_g^i (i = 1, 2, 3)$ of an empty E-plane Y-junction: $z = c/4$ and $f = 10$ GHz.

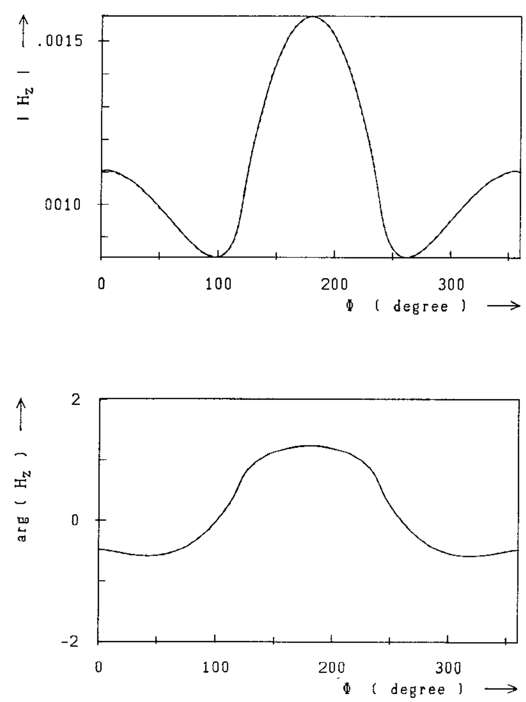

Fig. 4. Distribution of the z-component of the magnetic field at $S_g^i (i = 1, 2, 3)$ of an empty E-plane Y-junction: $z = c/4$ and $f = 10$ GHz.

genvalues corresponding to the scattering matrix. The solid lines show the results with the terms $\underline{G}_{oq}$ taken into account, and the broken lines show the results without these terms. The experimental results of [11] are represented by the triangles. Obviously for the correctness of the numerical results, the irrotational set has to include the terms $\underline{G}_{oq}$. Since these terms have constant values on the r-ϕ-plane, one can expect that only the argument of the nondegenerate eigenvalue φ_o will be affected as shown in Fig. 2 while the arguments of the degenerate eigenvalues $\varphi_\pm$ do not change.

The tangential magnetic field at S_g^i has ϕ- and z-components. The continuity of the tangential magnetic field is shown in Figs. 3 and 4. The solid lines represent the field in the waveguides just in front of S_g^i, and the broken lines represent the field just inside the junction. For the z-component of the magnetic field, the two curves are so close to each other that they cannot be distinguished in the figure.

To ensure the correctness of the numerical results, the completeness terms $\underline{G}_{oq}$ are included in the following calculations.

B. Numerical Results for the E-Plane Y-Junction Containing Full-Height Ferrite Post

The convergence of our results for an E-plane Y-junction containing an Y13A full-height ferrite post is shown in Fig. 5. It can be seen that 40 expansion functions in radial direction are sufficient for the calculation. The number of the expansion functions in circumferential direction and of the waveguide modes are 21 and 11, respectively. The improvement of the numerical results is less than 1% by using 141 expansion functions in circumferential direction and 41 waveguide modes.

The performance of an E-plane Y-junction containing an Y13A full-height ferrite post is calculated. Shown in Fig. 6 are the calculated insertion, reflection, and isolation losses for the ferrite post with diameter of 5.08 mm

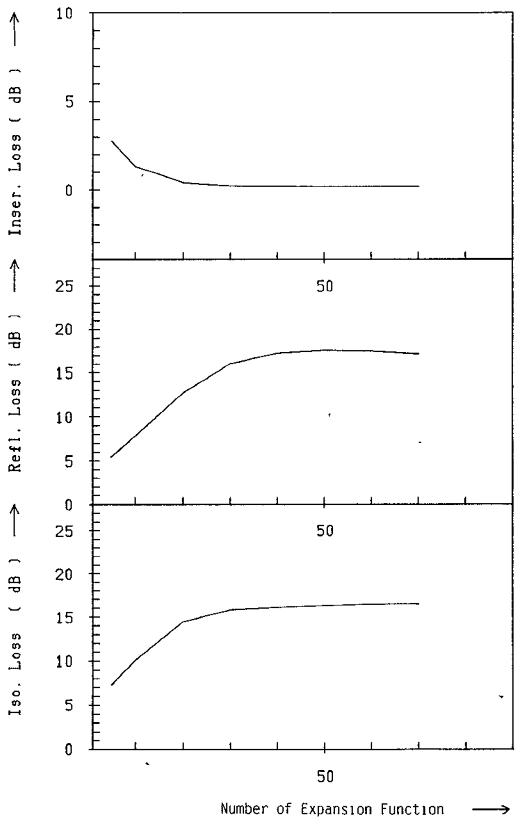

Fig. 5. Convergence behavior of the scattering matrix elements for an E-plane Y-junction with full-height Y13A ferrite post as a function of the number of expansion functions in radial direction: d = 5.08 mm and f = 11 GHz.

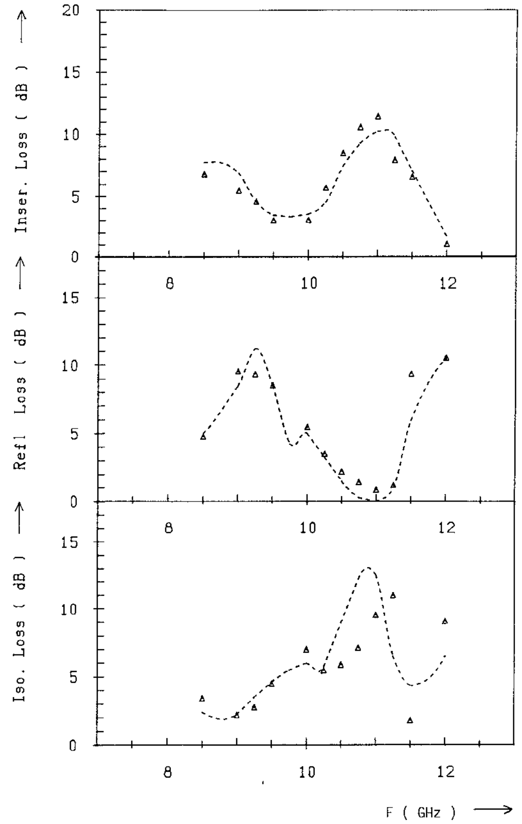

Fig. 7. Performance of an E-plane Y-junction with full-height Y13A ferrite post: d = 7.62 mm ---- measured values given by [1], △ values computed from this theory.

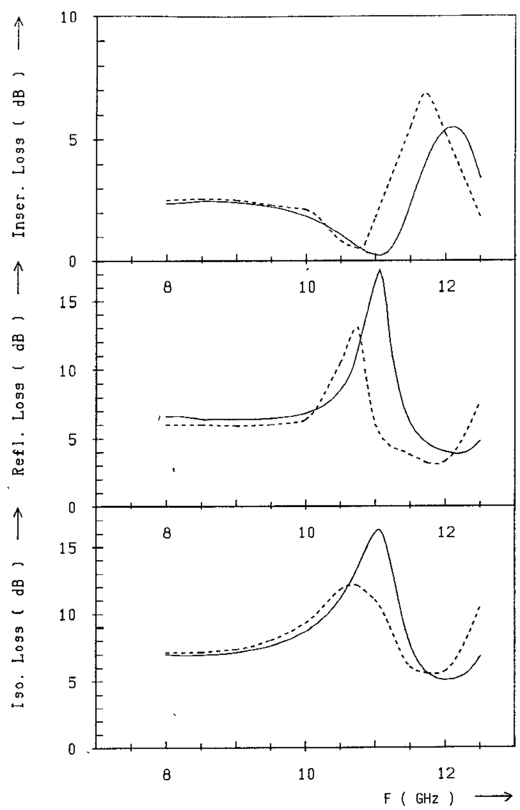

Fig. 6. Performance of an E-plane Y-junction with full-height Y13A ferrite post: d = 5.08 mm, ---- computed valued given by [1], ——— values computed from this theory.

and height of 22.86 mm. The numerical results of [1] by using the point-matching method are represented by the broken lines. The circulation frequency according to our results is at 11 GHz and according to the results of [1] at 10.6 GHz. The two results agree well in the range from 8 GHz to 10 GHz. Above 10 GHz, the two results are shifted at an amount of about 0.4 GHz and our calculated reflection and isolation losses are larger than those of [1] at the circulation frequency. Fig. 7 shows the computed circulator characteristics when using a ferrite post with diameter of 7.62 mm. The experimental results of [1] are represented by broken lines and our results are indicated by triangles. The two results also agree to a very good extent in this case. The internal dc magnetic field H_o used in all calculations is 200 Oe.

The electric-field distribution in the junction as a function of r at $z = c/4$ is shown in Fig. 8. The discontinuities of the electric field components E_ϕ and E_z at $r = a$ are due to the fact that magnetic currents are introduced there while the discontinuities at the ferrite-air interface are accompanied with Gibbs phenomenon.

Fig. 9 shows the power-density distribution in the junction as a function of ϕ at different values of r, for the case of d = 5.08 mm at a circulation frequency of 11 GHz and $z = c/4$. Specially, at r/a = 0.9 we can see that the power entering from the input port (ϕ = 120° to ϕ = 240°) is almost completely transmitted to the output port (ϕ = 240° to ϕ = 360°). The difference in sign of the

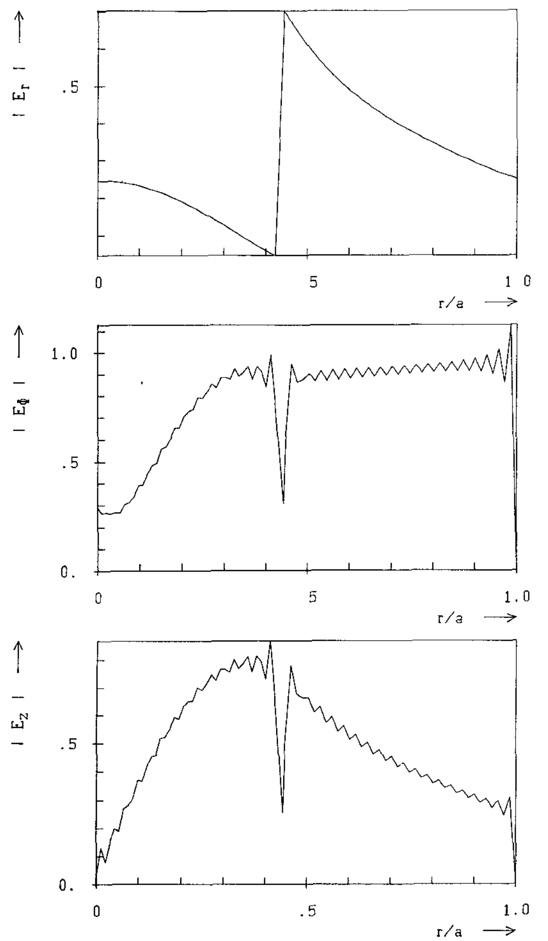

Fig. 8. Electric-field distribution in the junction of an E-plane Y-junction using a full-height Y13A ferrite post: d = 5.08 mm, $\phi = \pi$ and f = 11 GHz.

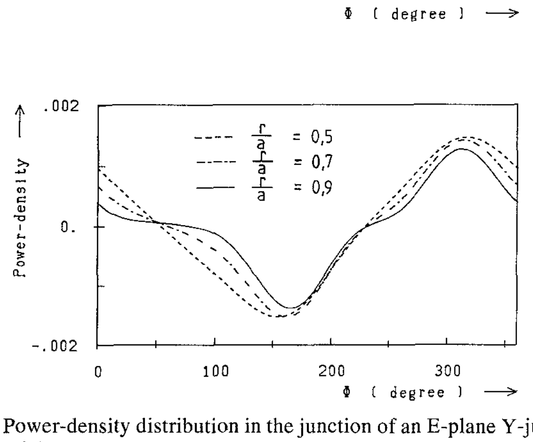

Fig. 9. Power-density distribution in the junction of an E-plane Y-junction using a full-height Y13A ferrite post: d = 5.08 mm and f = 11 GHz.

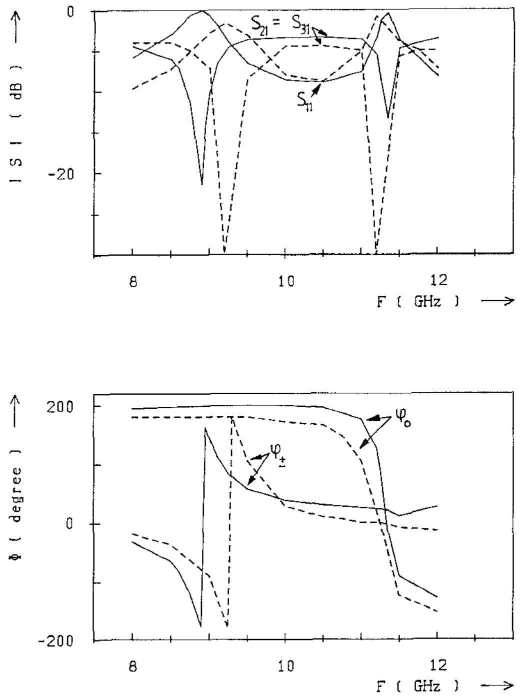

Fig. 10. S-parameters and arguments of the scattering matrix eigenvalues of an E-plane Y-junction with partial-height unmagnetized RF2 ferrite post: d = 8 mm, h = 5 mm, ϵ_r = 13.5 and H_o = 0. ———values computed from this theory, ---- measured values given by [11].

power-density is due to the fact that the power in the input port is incident to the junction while the power in the output port is leaving the junction.

C. Numerical Results for the E-Plane Y-Junction Containing Partial-Height Unmagnetized and Magnetized Ferrite Posts

If the dc magnetic field H_o is zero, the off-diagonal elements of the relative permeability tensor $\overleftrightarrow{\mu}_r$ are all zero while the diagonal elements have equal value which is somewhat smaller than 1 [12]. Fig. 10 shows the S-parameters and the arguments of the scattering matrix eigenvalues of an E-plane Y-junction with a partial-height unmagnetized RF2 ferrite post. Our results are represented by solid lines and the experimental results of [11] are indicated by broken lines. As can be seen the two results agree fairly good. The abrupt changes of the arguments of the eigenvalues at about 9.2 GHz and 11.2 GHz are due to the resonances of the $EH_{11 1/2}$ and $HE_{o1 1/2}$ modes of the unmagnetized ferrite post at these frequencies, respectively.

Fig. 11 shows the calculated S-parameters (solid lines) and the experimental results (broken lines) of [11] for an E-plane Y-junction with partial-height RF2 ferrite post. Diameter and height of the RF2 ferrite post are 8 mm and 5 mm, respectively. The agreement between the two results is not so good as for the results with unmagnetized ferrite post. The discrepancy may be due to the inhomogeneity of the dc magnetic field around the ferrite post which had about the same diameter as the permanent magnet used in the experiment. This inhomogeneity of the dc

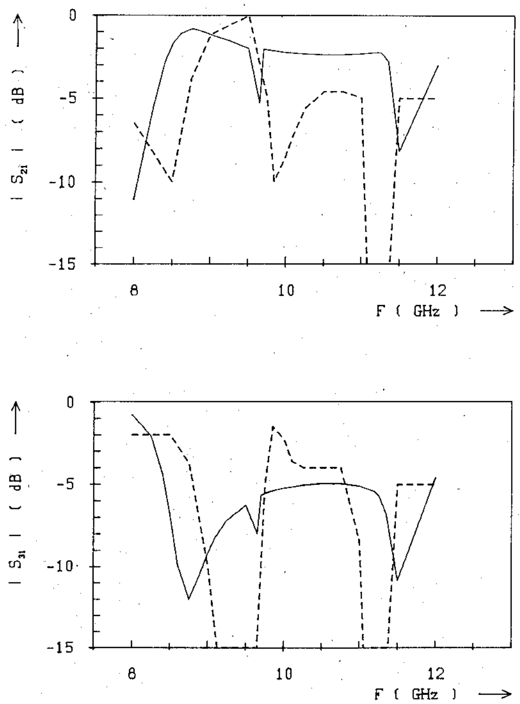

Fig. 11. Performance of an E-plane Y-junction with partial-height magnetized RF2 ferrite post. The parameters of the RF2 ferrite material are $d = 8$ mm, $h = 5$ mm, $\epsilon_r = 13.5$, $M_o = 185$ KA$/m$, $g = 2.03$, and $H_o = 200$ Oe, ---- measured values given by [11], ———values computed from this theory.

magnetic field around the ferrite post leads to values of the relative permeability tensor elements which differ from those used in the numerical calculations.

V. Conclusion

A method based on the equivalence principle and on cavity field expansions for the analysis of E-plane waveguide junctions with an anisotropic medium has been presented. It has been shown that in order to completely describe the fields in the junction the completeness terms $\underline{G}_{oq}$ have to be included in the irrotational set. The performance of the E-plane Y-junctions containing full-height and partial-height ferrite post has been calculated and compared with previously published theoretical and experimental results. The electric-field and power-density distributions in the junction have been plotted.

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Yu-Yuan Tsai was born in Taichung, Taiwan. He received the Dipl.-Ing. degree in electrical engineering from the Technische Universität Braunschweig, Germany, in 1988. Since 1989 he has been working towards the Ph.D. degree in the area of microwave.

From 1979–1981, he was with the Chinese Navy, Taiwan, as a Second Lieutenant. From 1981–1983, he was a Design Engineer with Cal-Comp Electronics, Inc., Taiwan, where he was engaged in the design of computer terminals. From January 1989 to May 1992 he was with the Institut für Hochfrequenztechnik, Technische Universität Braunschweig. Now, he is a Research Engineer in the Arbeitsbereich Hochfrequenztechnik, Technische Universität Hamburg-Harburg, Germany.

Abbas S. Omar (M'86–SM'89) received the B.Sc. and M.Sc. degrees in electrical engineering from Ain Shams University, Cairo, Egypt, in 1978 and 1982, respectively, and the Doktor-Ing. degree from the Technische Universität Hamburg-Harburg, Germany, in 1986.

From 1978 to 1982, he served as a Research and Teaching Assistant in the Department of Electronics and Computer Engineering of Ain Shams University, where he was engaged in investigation of microstrip lines and below-cutoff waveguides and their use in a hybrid circuit technique for the realization of broad-band tunable oscillators. From 1982 to 1983, he was with the Institut für Hochfrequenztechnik, Technische Universität Braunschweig, Germany, as a Research Engineer, where he was involved with theoretical investigation of finlines. From 1983 to 1987 he held the same position at the Technische Universität Hamburg-Harburg, where he was engaged in investigations of planar structures and dielectric resonators. From 1987 to 1990 he was a Senior Research Engineer at the Technische Universität Braunschweig. Since 1990 he has been a Professor of Electrical Engineering and head of the field theory group at the Arbeitsbereich Hochfrequenztechnik, Technische Universität Hamburg-Harburg. His current fields of research are concerned with optimization of microwave ovens, the field analysis of dielectric resonators, the analysis and design of three-dimensional passive systems, theoretical investigations of guiding structures, and the design and optimization of planar structures.